A. Introductory Problems

1. A polynomial equation can be written in terms of coe cients

$$f(x) = a_n x^n + a_{n\ 1} x^{n\ 1} + \quad + a_1 x + a_0 = 0$$

or in terms of roots

$$f(x) = a_n(x \quad _1)(x \quad _2) \quad (x \quad _n) = 0:$$

(a) By expanding these expressions show that

$$\sum_{k=1}^{n}$$

5. Express the following as partial fractions:

$$\text{(a)}\ \frac{2x+1}{x^2+3x-10} \qquad \text{(b)}\ \frac{4}{x^2-3x} \qquad \text{(c)}\ \frac{x^2+x-1}{x^2+x-2}$$

$$\text{(d)}\ \frac{2x}{(x+1)(x-1)^2} \qquad \text{(e)}\ \frac{2+4x}{(x+2)(x^2+2)}$$

(In case (c) you should start by dividing to find the quotient and remainder; in cases (d) and (e) you will need three constants.)

6. Use a binomial expansion to evaluate (a) $(1+x)^5$, (b) $1/(1+x)$, and (c) $1/\sqrt{1+x}$ up to second order. Use the last result to find a value of $1/\sqrt{4.2}$ to three decimal places.

7. Prove by induction that

   (a) $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

   (b) $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

8. The sum of a geometric progression can be written as

$$S_n = 1 + r + r^2 + \cdots + r^n$$

Evaluate rS_n and use this to show that $S_n = (1-r^{n+1})/(1-r)$.

B. Calculus

6. Find the first derivatives of $\sinh(x)$ and $\tanh(x)$ with respect to x, expressing your answers in terms of $\cosh(x)$.

Integration

7. Evaluate the following indefinite integrals: (a) $\int (1 + 2x + 3x^2)\,dx$, (b) $\int [\sin(2x) - \cos(3x)]\,dx$, (c) $\int (e^t + \frac{1}{t^2})\,dt$, (d) $\int \,dw$.

8. Evaluate the following definite integrals:

   (a) $\int_{-1/4}^{1/4} \cos(2\pi x)\,dx$

   (b) $\int_0^3 (2t - 1)^2\,dt$

   (c) $\int_1^2 \frac{(1+e^t)^2}{e^t}\,dt$

   (d) $\int_4^9 \sqrt{x}(x - \frac{1}{x})\,dx$

   (e) $\int_{-1}^1 x^3\,dx$; explain your answer with a sketch.

9. Find the indefinite integral $\int x^2 e^{-x}\,dx$.

10. Find the indefinite integral $\int \sin x(1 + \cos x)^4\,dx$.

11. Find the area of the region bounded by the graph of the function $y = x^2 + 2$ and the line $y = 5 - 2x$.

12. Although integrals are frequently thought of as areas und

C. Mechanics

Motion in one dimension

1. A car accelerates uniformly from rest to 80 km per hour in 10 s. How far has the car travelled?

2. A stone falls from rest with an acceleration of $9.8\,\mathrm{m\,s^{-2}}$. How fast is it moving after it has fallen through 2 m?

3. A car is travelling at an initial velocity of $6\,\mathrm{m\,s^{-1}}$. It then accelerates at $3\,\mathrm{m\,s^{-2}}$ over a distance of 20 m. What is its final velocity?

Work and energy

4. The brakes on a car of mass 1000 kg travelling at a speed of $15\,\mathrm{m\,s^{-1}}$ are suddenly applied so that the car skids to a stop in a distance of 30 m. Use energy considerations to determine the magnitude of the total frictional force acting on the tyres, assuming it to be constant throughout the braking process. What is the car's speed after the first 15 m of this skid?

5. The gravitational potential energy for a mass m at a distance $R + h$ from the centre of the earth (where R is the radius of the earth) is $-GMm/(R + h)$ where G is Newton's gravitational constant and M is the mass of the earth. If $h \ll R$ show that this is approximately equal to a constant (independent of h) plus mgh, where $g = GM/R^2$. [Hint: write $R + h = R(1 + h/R)$ and expand $(1 + h/R)^{-1}$ by the binomial theorem.]

6. Show that the minimum speed with which a body can be projected from the surface of the earth to enable it to just escape from the earth's gravity (and reach "infinity" with zero speed) is given by $v_{escape} = \sqrt{2GM/R}$, where G, M and R are defined as in the previous question. If a body is projected vertically with speed $\frac{1}{2}v_{escape}$, how high will it get? (Give the answer in terms of R and neglect air resistance throughout this question.)

Simple harmonic motion

7. The position of a particle as a function of time is given by
   $x(t) = A \sin(! \, t +$